# MECHANICS OF FLUIDS

Lecture 8 – Energy Equation

Lecturer: Hamidreza Norouzi

# Note

- All the art-work contents of this lecture are obtained from the following sources, unless otherwise stated:
  - *Fluid Mechanics, 8th edition, Frank M. White, McGraw-Hill, 2016.*
  - *Fluid Mechanics: Fundamental and Applications, 3rd edition, Yunus A. Cengel, John M. Cimbala, McGraw-Hill, 2014.*

# System analysis

- The first law of thermodynamics for a closed system
  - *The energy change of the system during a process is equal to the received heat added to system minus the work done by system.*

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt}$$

- Using Reynolds transport theorem, this can be expressed for a control volume:

$$\frac{d}{dt}(B_{\text{syst}}) = \frac{d}{dt}\left(\int_{\text{CV}} \beta\rho \, d\mathcal{V}\right) + \int_{\text{CS}} \beta\rho(\mathbf{V}_r \cdot \mathbf{n}) \, dA$$

$E$

$e = dE/dm$

Amirkabir University of Technology

# System analysis

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\left(\int_{CV} e\rho \, d\mathcal{V}\right) + \int_{CS} e\rho(\mathbf{V}\cdot\mathbf{n})\, dA \qquad (*)$$

Rate of heat added to system (through CS)

We leave this to Heat Transfer course

Rate of work done by system

$$e = e_{\text{internal}} + e_{\text{kinetic}} + e_{\text{potential}}$$

$$e = \hat{u} + \tfrac{1}{2}V^2 + gz$$

$$\dot{W}_{\text{shaft}} + \dot{W}_{\text{press}} + \dot{W}_{\text{viscous stresses}} = \dot{W}_s + \dot{W}_p + \dot{W}_v$$

Amirkabir University of Technology

# System analysis

- $\dot{W}s$ is energy rate out of control volume through shaft work, like pump, turbine, fan and etc.
- Pressure work:

$$d\dot{W}_p = -(p\,dA)V_{n,\,\text{in}} = -p(-\mathbf{V}\cdot\mathbf{n})\,dA$$

$$\boxed{\dot{W}_p = \int_{\text{CS}} p(\mathbf{V}\cdot\mathbf{n})\,dA}$$

- And similarly the shear work is:

$$\boxed{\dot{W}_v = -\int_{\text{CS}} \boldsymbol{\tau}\cdot\mathbf{V}\,dA}$$

Amirkabir University of Technology

# System analysis

- Substitution into the main equation (∗), we get:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\left(\int_{CV} e\rho \, d\mathcal{V}\right) + \int_{CS}\left(e + \frac{p}{\rho}\right)\rho(\mathbf{V}\cdot\mathbf{n})\, dA$$

$$e = \hat{u} + \tfrac{1}{2}V^2 + gz \qquad \hat{h} = \hat{u} + p/\rho$$

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \frac{\partial}{\partial t}\left[\int_{CV}\left(\hat{u} + \tfrac{1}{2}V^2 + gz\right)\rho d\mathcal{V}\right] + \int_{CS}\left(\hat{h} + \tfrac{1}{2}V^2 + gz\right)\rho(\mathbf{V}\cdot\mathbf{n})\, dA$$

# Energy equation of steady flow

- For a control volume at steady condition with one inlet and one outlet and uniform flow at inlet and outlet:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = -\dot{m}_1(\hat{h}_1 + \tfrac{1}{2}V_1^2 + gz_1) + \dot{m}_2(\hat{h}_2 + \tfrac{1}{2}V_2^2 + gz_2)$$

- And considering the fact that $\dot{m}_1 = \dot{m}_2 = \dot{m}$

$$\underbrace{\hat{h}_1 + \tfrac{1}{2}V_1^2 + gz_1}_{\text{Stagnation enthalpy at inlet}} = \underbrace{(\hat{h}_2 + \tfrac{1}{2}V_2^2 + gz_2)}_{\text{Stagnation enthalpy at outlet}} - q + w_s + w_v$$

$$q = \dot{Q}/\dot{m}$$

$$w_s = \dot{W}_s/\dot{m}$$

$$w_v = \dot{W}_v/\dot{m}$$

What does this equation mean?

# Energy equation of steady flow

- If we divide this equation by g, the dimension of each term becomes length [m or ft]:

$$\frac{p_1}{\gamma} + \frac{\hat{u}_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\hat{u}_2}{g} + \frac{V_2^2}{2g} + z_2 - h_q + h_s + h_v$$

Amirkabir University of Technology

# For a pipe system with incompressible flow

- Assuming no heat transfer with surroundings ($h_q$=0).
- The friction loses exist in the control volume, they irreversibly convert the mechanical energy to internal energy.

$$\frac{\hat{u}_2 - \hat{u}_1}{g} = h_{friction}$$

- Assuming negligible viscous work at inlet and outlet ($h_v$ = 0).
- The shaft work can be either pump work on fluid (negative in our convention) or turbine work (positive in our convention)

$$h_s = -h_{pump} + h_{turbine}$$

# For a pipe system with incompressible flow

$$\underbrace{\left(\frac{p}{\gamma}+\frac{V^2}{2g}+z\right)_{\text{in}}}_{\text{Total head at inlet}} = \underbrace{\left(\frac{p}{\gamma}+\frac{V^2}{2g}+z\right)_{\text{out}}}_{\text{Total head at outlet}} + h_{\text{friction}} - h_{\text{pump}} + h_{\text{turbine}}$$

Friction/head loss

Head added by pump

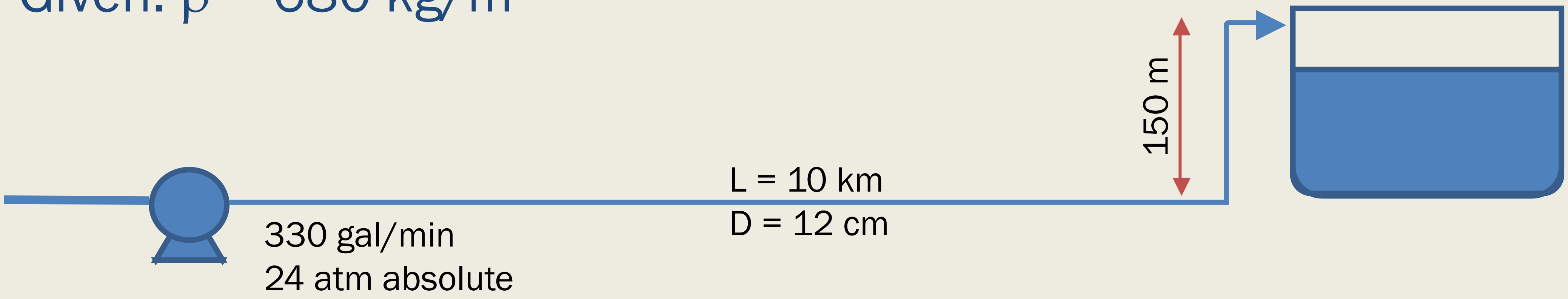
Head produced by turbine

# Example 1

Gasoline at 20 °C is pumped through a smooth 12-cm-diameter pipe 10 km long, at a flow rate of 75 $m^3/h$ (330 gal/min). The inlet is fed by a pump at an absolute pressure of 24 atm. The exit is at standard atmospheric pressure and is 150 m higher. Estimate the frictional head loss $h_f$ , and compare it to the velocity head of the flow.

Answer: $h_f = 199$ m

Given: $\rho = 680$ kg/$m^3$




Amirkabir University of Technology

# Example 1

- Energy equation with $h_{pump} = h_{turbine} = 0$

$$\frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} = \frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} + h_f$$

- Average inlet and outlet velocities:

$$V_{\text{in}} = V_{\text{out}} = \frac{Q}{A} = \frac{Q}{(\pi/4)D^2} = \frac{(75\ \text{m}^3/\text{h})/(3600\ \text{s/h})}{(\pi/4)(0.12\ \text{m})^2} \approx 1.84\ \frac{\text{m}}{\text{s}}$$

$$\frac{(24)(101{,}350\ \text{N/m}^2)}{6670\ \text{N/m}^3} + 0.173\ \text{m} + 0\ \text{m} = \frac{101{,}350\ \text{N/m}^2}{6670\ \text{N/m}^3} + 0.173\ \text{m} + 150\ \text{m} + h_f$$

$$\boxed{h_f = 364.7 - 15.2 - 150 \approx 199\ \text{m}}$$

Amirkabir University of Technology

# Kinetic energy correction factor

- Recall that in energy equation (steady condition) there is a term containing fluid velocity:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \int_{CS} \left( \hat{h} + \tfrac{1}{2}V^2 + gz \right) \rho (\mathbf{V} \cdot \mathbf{n})\, dA$$

- We obtained the following equation by assuming an average uniform flow at inlet and outlet:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = -\dot{m}_1(\hat{h}_1 + \tfrac{1}{2}V_1^2 + gz_1) + \dot{m}_2(\hat{h}_2 + \tfrac{1}{2}V_2^2 + gz_2)$$

# Kinetic energy correction factor

- This assumption can be erroneous and should be modified by inserting a correction factor into the energy equation as follows:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = -\dot{m}_1(\hat{h}_1 + \alpha\,\tfrac{1}{2}V_1^2 + gz_1) + \dot{m}_2(\hat{h}_2 + \alpha\,\tfrac{1}{2}V_2^2 + gz_2)$$

$$\int_{\text{port}} (\tfrac{1}{2}V^2)\rho(\mathbf{V}\cdot\mathbf{n})\,dA \equiv \alpha(\tfrac{1}{2}V_{\text{av}}^2)\dot{m} \qquad V_{\text{av}} = \frac{1}{A}\int u\,dA$$

- If $u$ is the velocity normal to the surface:

$$\tfrac{1}{2}\rho\int u^3 dA = \tfrac{1}{2}\rho\alpha V_{\text{av}}^3 A \Longrightarrow \alpha = \frac{1}{A}\int\left(\frac{u}{V_{\text{av}}}\right)^3 dA$$

Amirkabir University of Technology

# Kinetic energy correction factor

- For laminar flow:

$$u = U_0\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$V_{\mathrm{av}} = 0.5U_0$$

$$\Longrightarrow \quad \alpha = 2.0$$

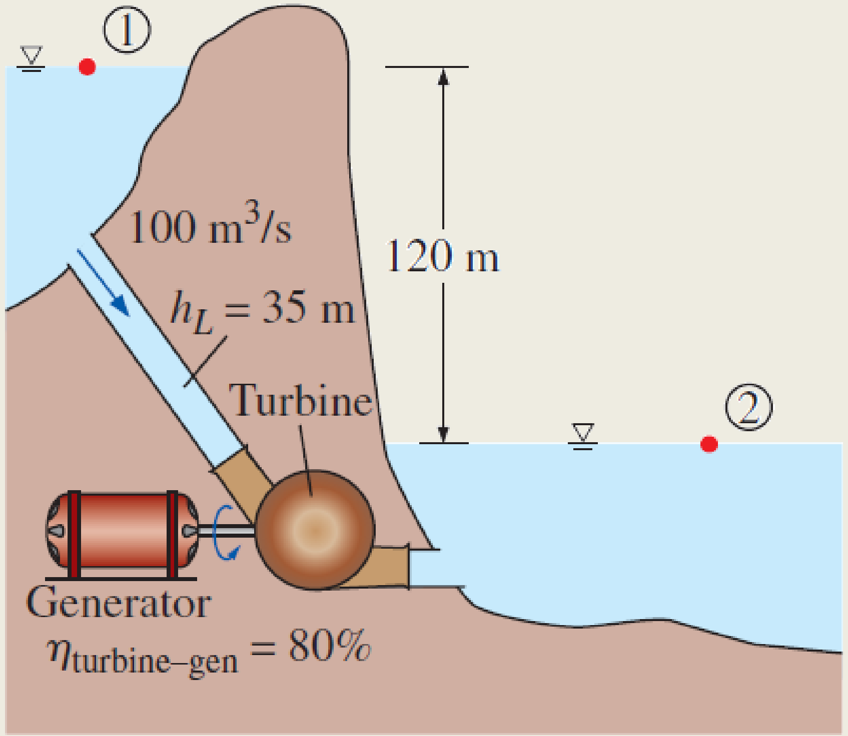
- For turbulent flow:

$$u \approx U_0\left(1 - \frac{r}{R}\right)^m \quad \Longrightarrow \quad \alpha = \frac{(1+m)^3(2+m)^3}{4(1+3m)(2+3m)}$$

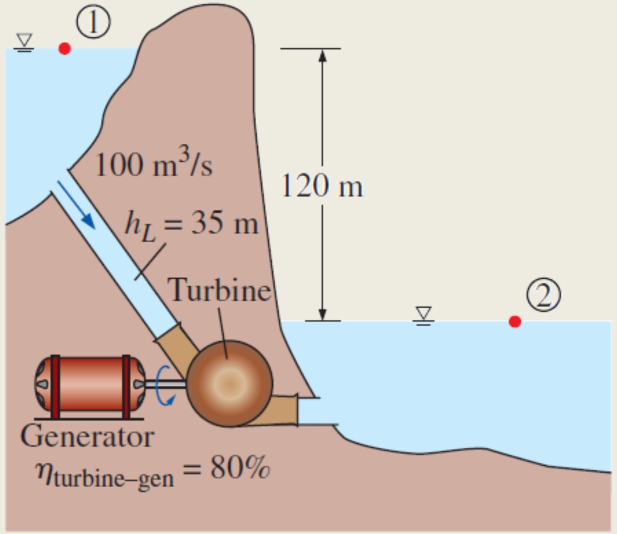
| $m$ | $\frac{1}{5}$ | $\frac{1}{6}$ | $\frac{1}{7}$ | $\frac{1}{8}$ | $\frac{1}{9}$ |
|---|---|---|---|---|---|
| $\alpha$ | 1.106 | 1.077 | 1.058 | 1.046 | 1.037 |

# Example 2

In a hydroelectric power plant, 100 $m^3/s$ of water flows from an elevation of 120 m to a turbine, where electric power is generated. The total irreversible head loss in the piping system from point 1 to point 2 is 35 m. If the overall efficiency of the turbine–generator is 80%, estimate the electric power output.



# Example 2




- Energy equation between points 1 and 2:
  - *Velocities are negligible.*
  - *$P_1$ and $P_2$ are cancelled out.*

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \cancel{\frac{V_1^2}{2g}} + z_1 + h_{\text{pump},\,u} \nearrow 0 = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \cancel{\frac{V_2^2}{2g}} + z_2 \nearrow 0 + h_{\text{turbine},\,e} + h_L$$

$$\Rightarrow \quad h_{\text{turbine},\,e} = z_1 - h_L = 120 - 35 = 85 \text{ m}$$

# Example 2

- The mechanical work of turbine:

$$\dot{W}_{\text{turbine, }e} = \dot{m}gh_{\text{turbine, }e} = (10^5\ \text{kg/s})(9.81\ \text{m/s}^2)(85\ \text{m})\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = 83{,}400\ \text{kW}$$

- The electrical output of generator

$$\dot{W}_{\text{electric}} = \eta_{\text{turbine–gen}}\dot{W}_{\text{turbine, }e} = (0.80)(83.4\ \text{MW}) = \mathbf{66.7\ MW}$$

Amirkabir University of Technology